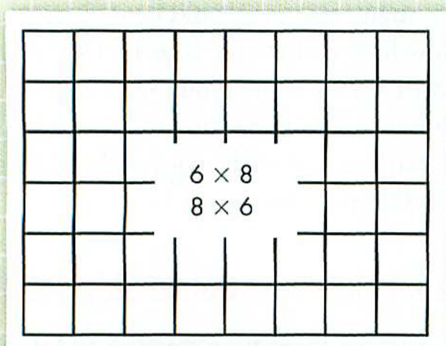
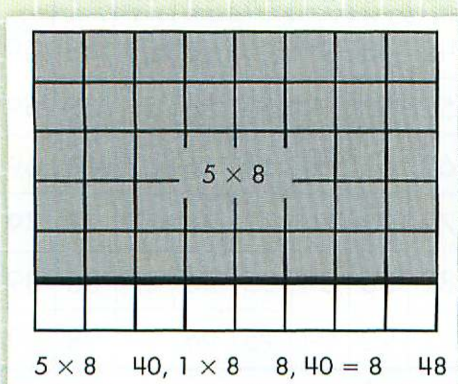
Representing Multiplication with Arrays

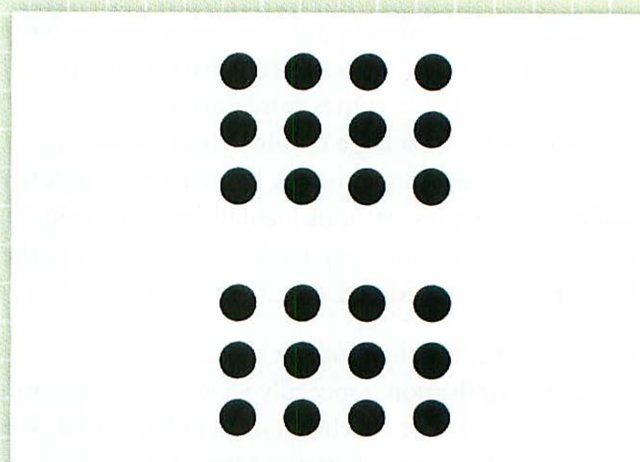
In this unit, students work with Array Cards and drawings in which all the individual units of the array are visible. These arrays are a representation of the groups and amounts in a group in any multiplication problem.



This 6 × 8 array can be seen as 8 groups of 6 items or as 6 groups of 8 items. In either case, students can visualize the problem as a whole and then visualize the smaller parts that may help them find the product. For example, if juice boxes come in sets of 6, a student might think of 8 × 6 as 8 sets of juice boxes. The student could visualize these in an array and use that image to break the problem into parts that are easier to solve, as follows:



The story problems in Investigations 1 and 2 and the *Quick Images* activity that is introduced in this unit (page 59) also provide experience with arrays that represent multiplication situations. For example, *Student Activity Book* page 1 shows cans in a 6 × 8 array. The *Quick Images* in this unit show combinations of arrays that provide the opportunity to describe a product in different ways. For example, *Quick Image* 7 might be described as 6 × 4 (6 rows of 4 dots), 8 × 3 (8 groups of vertical rows of 3 dots each), 2 × 12 (2 rectangles, each with 12 dots), or even 2 × 3 × 4 (2 rectangles, each with 3 rows of 4 dots).



Visualizing how to break multiplication problems into parts becomes even more important as students solve multidigit problems in *Multiple Towers and Division Stories* and *How Many Packages? How Many Groups?* See **Teacher Note:** Representing Multiplication with Arrays (page 117), for more information about how arrays are used in this unit and how the use of arrays can be extended to represent more difficult multiplication and division problems. In the next unit on multiplication and division, *Multiple Towers and Division Stories,* the **Teacher Note:** Visualizing Arrays provides information about how and why students make a transition from using arrays marked with individual units to visualizing unmarked arrays.

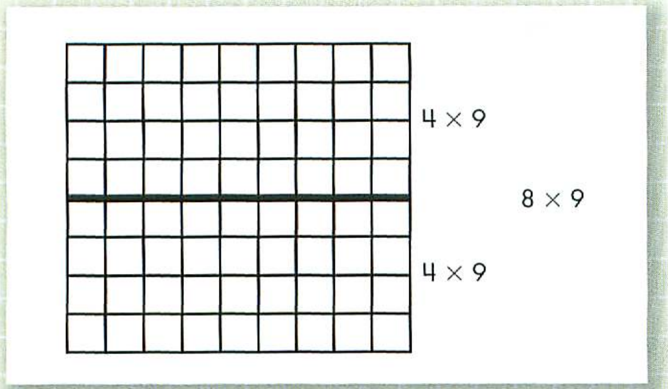
Teacher Note

Representing Multiplication with Arrays

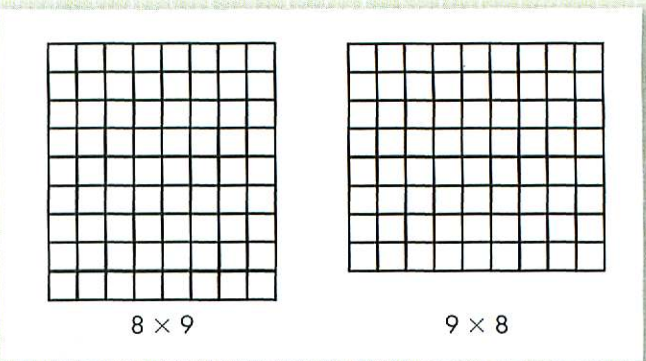
Representing mathematical relationships is a key element of developing mathematical understanding. For multiplication, the rectangular array is an important tool. It meets all the criteria for a powerful mathematical representation: it highlights important relationships, provides a tool for solving problems, and can be extended as students apply ideas about multiplication in new areas.

Why Arrays for Multiplication?

As students come to understand the operation of multiplication in Grades 3 and 4, they gradually move away from thinking of multiplication as only repeated addition. They learn that multiplication has particular properties that distinguish it from addition. Although a number line or 100 chart can be used to show how multiplication can be viewed as adding equal groups, neither of these tools provides easy access to other important properties of multiplication. The rectangular array provides a window into properties that are central to students' work in learning the multiplication combinations and in solving multidigit multiplication and division problems.

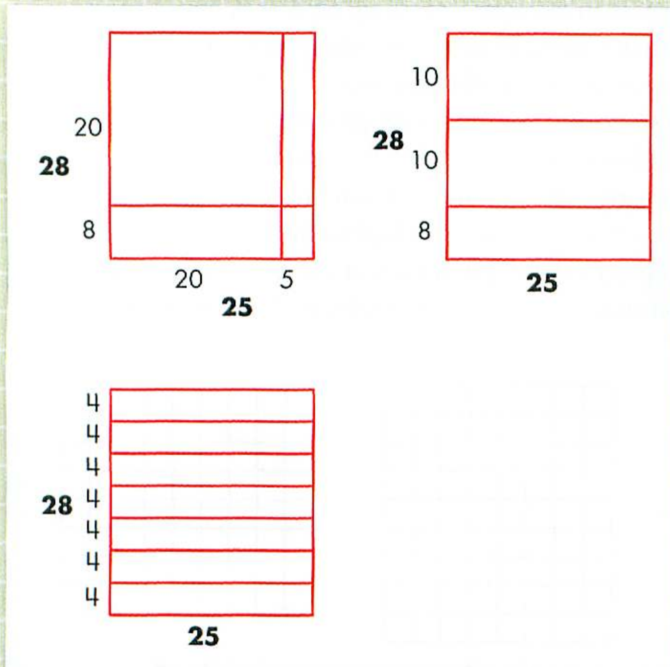


Here is an example—working on 8×9, one of the more difficult multiplication combinations for most students—of how an array can illustrate that $8 \times 9 = (4 \times 9) + (4 \times 9)$. In splitting a multiplication problem such as 8×9 into sub problems $[(4 \times 9) + (4 \times 9)]$, you are using the distributive property. The number that you break up is distributed into parts that must all be multiplied by the other number. This property of multiplication is at the core of almost all common strategies used to solve multiplication problems.

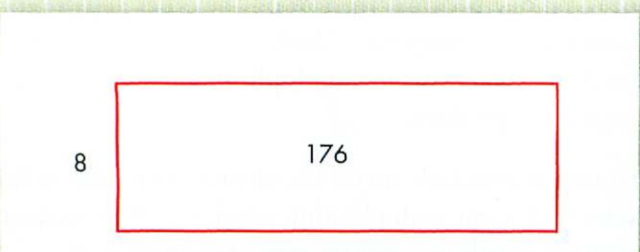


The rectangular array also makes it clearer why the product of 9×8 is the same as the product of 8×9. The array can be rotated to show that 9 rows with 8 in each row have the same number of squares as 8 rows with 9 in each row. The column on one becomes the row on the other, illustrating the commutative property—the fact that you can change the order of the factors in a multiplication equation without changing the product.

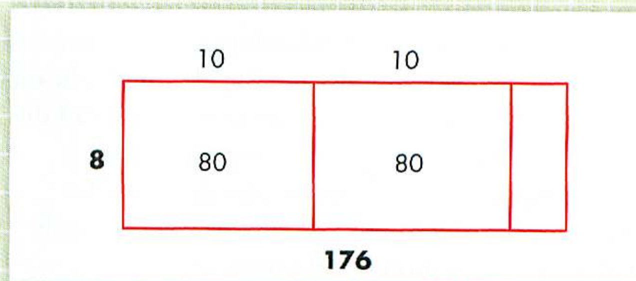
Arrays are particularly useful for solving or visualizing how to solve multidigit multiplication problems. After students have worked with rectangular arrays for single-digit multiplication combinations and thoroughly understand how an array represents the factors and product, they can use arrays in their work to solve harder problems later in Grade 4. For example, the array for 28 × 25 can be broken up in many ways, as follows:



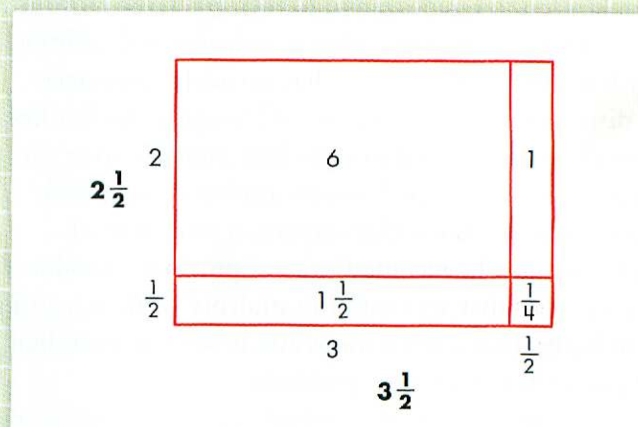
Arrays also support students' learning about the relationship between multiplication and division. In a division problem such as 176 ÷ 8, the dividend (176) is represented by the number of squares in the array, and the divisor (8) is one dimension of the array.

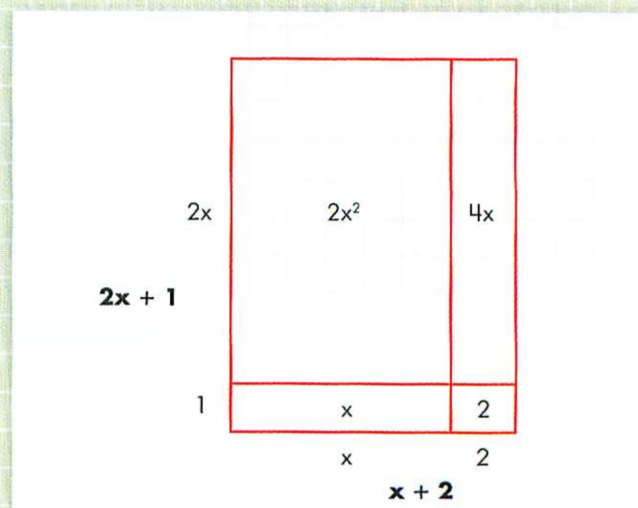


Students can think of "slicing off" pieces of the rectangle as they gradually figure out the other factor:

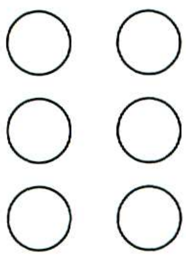


Finally, the use of the rectangular array can be extended in later grades as students work with multiplication of fractions and, later, of algebraic expressions.





For multiplication notation to describe arrays, the *Investigations* curriculum uses the convention of designating the number of rows first and the number in each row second; for example, 3 × 2 indicates 3 rows with 2 in each row.

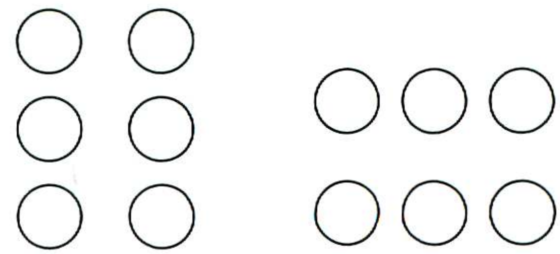


3 × 2

"There are 3 rows with 2 circles in each row."

This convention is consistent with using 3 × 2 to indicate 3 groups of 2 in other multiplication situations (e.g., 3 pots with 2 flowers in each pot). However, at this age level, it is not necessary for students to follow this system rigidly; trying to remember which number stands for rows and which for the number in a row can be unnecessarily distracting for students.

When students suggest a multiplication expression for an array, what is important is that they understand what the numbers mean; for example, a student might show how 3 × 2 represents 3 rows of cans with 2 in each row or 3 cans in each of 2 rows.



3 × 2

"There are 3 rows with 2 circles in each row."

3 × 2

"See, there are 3 here and 3 here, so you double the 3."

Note that in other cultures, conventions about interpreting multiplication expressions differ. In some countries, the convention for interpreting 3 × 2 is not "3 groups of 2" but "3 taken 2 times."